SCATTERING REGIMES: (Depends on ratio λ/L)

High Frequency: λ<<L, Optics like scattering, mostly independent scattering centers, angle of incidence = angle of reflection (backscatter, surface normal points back toward radar); Signature is coherent sum (phasor addition) of scattering centers.

Coherent Sum: $\sigma = \left| \sum_{i=1}^{N} \sqrt{\sigma_i} \exp(-j2\mathbf{k}\cdot\mathbf{r}_i) \right|^2$

Incoherent Sum: $\sigma = \sum \sigma_i$

Resonant Region: λ ~ L, Surface traveling, edge, and creeping waves become important scattering mechanisms. Must have component of E in direction of propagation. Grazing angle phenomena. Max amplitude at

$$\theta_{max} = 49\sqrt{\lambda / L} \text{ degrees}$$

Maximum amplitude depends on aft reflection coefficient and surface impedance, usually less than $3\lambda^2$ for PECs.

Low Frequency (Rayleigh Region): λ >> L, Induced dipole moment. Scattering proportional to (frequency)^4:

$$\sigma \approx (4/\pi)k^4 V^2$$

CONSTANTS:

Permittivity:

$\varepsilon_0 = 8.85 \times 10^{-12}$ Farads / meter

$\varepsilon_r = \varepsilon / \varepsilon_0 = \varepsilon' - j\varepsilon''$, $\varepsilon'' = \sigma / (\omega\varepsilon_0)$

Permeability: $\mu_0 = 4\pi \times 10^{-7}$ Henrys/meter; $\mu_r = \mu / \mu_0$

Free space wave impedance:

$$\eta = E / H = \sqrt{\mu_0 / \varepsilon_0} = 120\pi = 377 \text{ ohms}$$

Velocity of Propagation: $c = (\varepsilon_0 \mu_0)^{-1/2} \approx 3 \times 10^8$ m/s

Index of Refraction: $n = \sqrt{\varepsilon_r \mu_r}$

Radian Frequency: $\omega = 2\pi f$

Wave Vector: Points in direction of propagation, $|\mathbf{k}| = 2\pi / \lambda$

Frequency and Wavelength relationships:

$f\lambda = c$; $\omega = ck$; $f_{MHz} = (300) / \lambda_{meter}$

$\omega\mu_0 = k\eta = 240\pi^2 / \lambda = 2367 / \lambda$;

$\omega\varepsilon_0 = k / \eta = 1/(60\lambda)$

MATERIAL CHARACTERIZATION:

Bulk Impedance: $Z = \eta\sqrt{\mu_r / \varepsilon_r}$

Reflection coefficient depends on polarization. For normal incidence it is

$R = (Z - \eta)/(Z + \eta)$; dB = 20 $\log_{10}$(R)

Surface impedance of material backed by ground plane, normal incidence:

General Case: $Z = \eta\sqrt{\mu_r / \varepsilon_r}\ \text{Tanh}(jk\sqrt{\varepsilon_r \mu_r}\ d)$

Thin layer: $Z = j\,\eta\,\mu_r\,k\,d$, independent of ε_r.

Resistive Layer, Ohms per Square: $R = 1/(\sigma\,\tau) = \rho / \tau$

Impedance of thin layer, Ohms per Square:

$Z = -j\eta / [k(\varepsilon_r - 1)\tau]$

Bulk Loss Characterization: $\varepsilon'' = \sigma / (\omega\varepsilon_0) = 60\lambda\sigma = 60\lambda / \rho$

MEASUREMENTS:

Radar Equation: $P_r = P_t G_t G_r \lambda^2 \sigma / (4\pi)^3 R^4$

Noise Power: $P_n(dBm) \approx -114 dBm + 10 \log B_{MHz} + NF_{db}$

Ground Plane Range: $h_{antenna} h_{target} = R\lambda / 4$;

Ideal peak gain = 16 = 12dB;

Far Field distance = $2D^2 / \lambda$.

Down Range Image:

Resolution $\Delta r = c/(2B) = (\lambda/2)/(\Delta f / f)$,

Extent = N Δ r

Cross Range Image: (isar)

Resolution $\Delta r = \lambda / (2\Delta\theta)$

DATA TYPES:

Probability Density Function (PDF): Probability P(σ) that σ lies between σ and σ + dσ (histogram).

Cumulative Distribution Function (CDF): $CDF(\sigma) = \int_{-\infty}^{\sigma} P(\sigma)d\sigma$

Median: $CDF(\sigma_m) = 0.5 = \int_{-\infty}^{\sigma_m} P(\sigma)d\sigma$

Geometric Average: $\sigma_g = [\prod_{i=1}^{N} \sigma_i]^{1/N} = 1/N \sum_{i=1}^{N} 10 \log_{10} \sigma_i$

Arithmetic Average: $\sigma_a = 1/N \sum_{i=1}^{N} \sigma_i$; usually $\sigma_a > \sigma_g$

Sector Average: Average over specified angular region

Cumulative Average: Average over increasing angular sector. At each point the cumulative average is that of the defined angular sector.

Window/Slide: Sliding window for averaging, e.g., 5° window, 1° slide

Units: Square Meters	dBsm	
1000	30	$\sigma_{dBsm} = 10 \log_{10} \sigma_{sm}$
100	20	
10	10	$\sigma_{sm} = 10^{**}(\sigma_{dBsm} / 10)$
1	0	
0.1	-10	
0.01	-20	

ANALYTICS:

Specular Reflection: $\theta_r = \theta_i$

Transmission (Snell's Law): $k_{inc} \sin\theta_{inc} = k_{tran} \sin\theta_{tran}$

Geometric Optics (specular scattering):

$\sigma_{go} = \pi R_1 R_2$, R_1 & R_2 radii of curvature at specular point

$$E^s = R\sqrt{\rho_1\rho_2 / ((\rho_1 + s)(\rho_2 + s))}\ \mathbf{D}\cdot\mathbf{E}^{inc} \exp(-jks)$$

Physical Optics (specular and end region scattering):

current: $\mathbf{J}_{po} = \begin{cases} 2\hat{\mathbf{n}} \times \mathbf{H}^{inc} & \text{illuminted} \\ 0 & \text{shadowed} \end{cases}$

Field: $\mathbf{H}^{scat} = \int (\mathbf{J}_{po} \times \nabla g)\, dS$

Electric and Magnetic Field Integral Equations:

$$\theta\mathbf{E}^{tot} = \mathbf{E}^{inc} + \int \left\{ (-jk\eta\,\mathbf{J}_e g) - (\mathbf{J}_m \times \nabla_s g) + \frac{\rho_e}{\varepsilon}\nabla_s g \right\} dS$$

$$\theta\mathbf{H}^{tot} = \mathbf{H}^{inc} + \int \left\{ (-j\frac{k}{\eta}\mathbf{J}_m g) + (\mathbf{J}_e \times \nabla_s g) + \frac{\rho_m}{\mu}\nabla_s g \right\} dS$$

$\theta = 1$ for $r \in R_i$; $1/2$ for $r \in \partial R_i$; 0 otherwise

Current Continuity: $\nabla \bullet \mathbf{J} = -\partial\rho / \partial t = -j\omega\rho$

Green's Functions:

3D:
$$\begin{cases} g = \exp(-j\mathbf{k}\cdot\mathbf{R}) / 4\pi R \\ \nabla g = (1 - jkR)\,\mathbf{R}\, g / R^2 \end{cases}$$

2D:
$$\begin{cases} g = \dfrac{1}{4j} H_0^{(2)}(k\rho) \\ \nabla g = \dfrac{k}{4j} H_1^{(2)}(k\rho)\,\hat{\boldsymbol{\rho}} \end{cases}$$

Method of Moments: $Z_{ij} I_j = V_i$; **W** = weight; **J** = basis

$$Z_{ij} = < \mathbf{W}_i, L(\mathbf{J}_j) >$$

$$= +jk\eta \iint [\mathbf{W}_i \bullet \mathbf{J}_j - (\nabla \bullet \mathbf{W}_i)(\nabla \bullet \mathbf{J}_j) / k^2]\, g\, dS_i\, dS_j$$

$V_i = < \mathbf{W}_i, \mathbf{E}^{inc} >$, $\mathbf{E}^{inc} = \mathbf{u}^{\theta \text{ or } \phi} E_0 \exp(-j\mathbf{k}\cdot\mathbf{R})$

$R_i^{\theta or \phi}$ = row measurement vector = $< \mathbf{J}_i, \mathbf{E}^{\theta \text{ or } \phi} >$

Unit Vectors:
$$\mathbf{u}^{\theta} = [\cos(\theta)\cos(\phi), \cos(\theta)\sin(\phi), -\sin(\theta)]$$
$$\mathbf{u}^{\phi} = [-\sin(\phi), \cos(\phi), 0]$$
$$\mathbf{k} = [-\sin(\theta)\cos(\phi), -\sin(\theta)\sin(\phi), -\cos(\phi)]$$

HIP POCKET ESTIMATION FORMULAS: (for backscatter, λ<<L)

Scattering Mechanisms:
Specular, end region, leading and trailing edge diffraction, surface traveling, creeping, and edge waves, multiple bounce, tip diffraction, etc.

Constant Phase Region (specular return):
$$\sigma_{specular} = 4\pi(\textit{Effective Area})^2 / \lambda^2$$

Characteristic Dimension of Constant Phase on Curved Surface:
$$\mathbf{L}^{effective} = \sqrt{(R\lambda / 2)}$$

where R = radii of curvature at specular point

Specular (Peak) Returns (polarization independent):

Planar Surfaces: $\sigma = 4\pi A^2 / \lambda^2$

Singly Curved Surface: $\sigma = kRL^2$

Doubly Curved Surface: $\sigma = \pi R_1 R_2$, R's at specular point

Approximate Beam Width: $\Delta\theta \approx 57\lambda / L$ (rect. distribution)

Flat Plate Formulas (sides a, b, or L):
$$\sigma = [4\pi(ab)^2 / \lambda^2]\cos^2(\theta)[\sin(P)/P]^2[\sin(Q)/Q]^2$$

where $P = ka\cos(\phi)\sin(\theta)$, $Q = kb\sin(\phi)\sin(\theta)$

Envelope ⊥ to Edge of Length L: $\sigma = L^2 \cot^2\theta / \pi$

Envelope Along Diagonal (45°): $\sigma = \lambda^2\cos^2\theta / (\pi^3\sin^4\theta)$

Edge Diffraction:
Leading Edge E parallel, Trailing Edge E perpendicular:
$$\sigma \approx L^2 / \pi$$

Curved Edge: $\sigma \approx R\lambda / 2\pi$

Corner Reflectors:

Dihedral: $\sigma = 8\pi(ab)^2 / \lambda^2$

Trihedral: $\sigma = 12\pi b^4 / \lambda^2$

Surface Traveling and Edge Wave (component of E in direction of propagation on surface):

Location: $\theta \approx 49(\lambda / L)^{1/2}$

Amplitude: Depends on aft reflection coefficient and surface impedance along propagation path, usually $\leq 3\lambda^2$

RCS of HOLES and SLOTS:

Use Babinet's principle: Interchange E & H, use complementary geometry (e.g., slot to wire, hole to disk), then use existing analytical approaches and computer codes for Rayleigh, resonant, or optical regimes.

Small holes of radius a, ka < 1:

$E_{\perp}$ plane of hole: $\sigma \approx (16 / 9\pi)a^2(ka)^4[2 + \sin^2\theta]^2$

$E_{\parallel}$ plane of hole: $\sigma \approx (4^3 / 9\pi)a^2(ka)^4\cos^4\theta$

Slots: Traveling wave for H in direction of slot

Complementary Impedance: $Z_{comp} = 377^2 / (4Z)$

Remember: RCS from arrays of holes or slots will phasor add & subtract. Peak σ when ⊥ to line of holes or slot

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RADAR CROSS SECTION REFERENCE

Marietta Scientific, Inc.

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RADAR CROSS SECTION σ: A measure of power reflected by a target. Units are square meters (area). When expressed in dB, the reference is 1 square meter, dBsm or dBm². Make non-dimensional by normalizing to wavelength, σ/λ^2.

MONOSTATIC CROSS SECTION: RCS in the backscatter direction, i.e., receiver at same location as transmitter. Usual case.

BISTATIC CROSS SECTION: RCS in direction other than backscatter, i.e., receiver at different location than transmitter.

POLARIZATION: Direction of E vector for transmit or receive. Typically vertical or horizontal.

WAVE VECTOR k: Vector direction of propagation for EM wave. Scalar magnitude inversely related to wavelength, k = 2π/λ.

EM WAVE: Propagation of electromagnetic energy. Has electric and magnetic vector components, **E** and **H**, and direction of propagation **k**. E, H, and k are mutually orthogonal. EM wave characterized by: wavelength or frequency, direction of propagation, and polarization of **E**. Wave impedance η = E/H = 120π = 377 ohms in free space.

SPECULAR POINT, FLASH POINT, REGION OF STATIONARY PHASE: Point on scattering body where angle of incidence is equal to angle of reflection. For back scatter, this is where the surface normal points back toward the radar.

SCATTERING CENTER: Region of body which reflects EM energy (hot spot), e.g. specular points, multiple reflection, surface wave, or diffraction locations.

SURFACE WAVE: Non specular scattering mechanism dominant for resonant region bodies, L/λ < 10. Types are traveling, creeping, or edge wave which occur near grazing incident angles when the incident E field has a vector component along the body in direction of propagation.

BAND	FREQUENCY	WAVELENGTH
HF	5-30 MHz	200-33 ft.
VHF	50-300 MHz	18-3 ft.
UHF	300-1000 MHz	3-1 ft.
L	1-2 GHz	1-0.5 ft.
S	2-4 GHz	6-3 in.
C	4-8 GHz	3-1.5 in.
X	8-12.5 GHz	1.5-0.9 in.
K_u	12.5-18 GHz	0.9-0.66 in.
K_a	26.5-40 GHz	0.45-0.3 in.